Ground Voltage and Current Cancellation by Co-axial Cable.

Garth W. Milne

Abstract— **The most frequent use of co-axial cable is to prevent electrical noise entering signals being passed between equipment. Ground voltage differences are common and add in series with the signal voltage. The screen of co-axial cable grounded at both ends induces a voltage in the inner conductor which cancels higher frequency ground voltage differences, starting from a break point of typically 2 kHz. The signal current also produces a flux external to the screen, which induces a loop current through the screen and earth path which opposes the earth current. Signal currents above 2 kHz thus increasingly return through the screen, in spite of a lower impedance ground path. An equivalent circuit explains the phenomena and the extreme sensitivity of co-ax and oscilloscope probes to short pigtail leads. Experiments are described and data presented which justify the theory. Remarks are made on end connections, cable trays, and feeding power lines via co-ax.**

Keywords— **Co-axial cable, earth voltage, earth current, pigtail, EMC**

I. Introduction

THE most common use of co-axial cable is to prevent cross-coupling of signals passing between equipment. Simple theory which gives good insight into the cross-coupling is seldom taught at universities, so few engineers understand the interactions of grounding, pigtail leads, and earth currents when using co-axial cable. This paper reviews the theory, discusses its consequences, describes simple experiments to demonstrate the phenomena, and suggests an unusual application of co-ax cable in equipment.

A. Ground voltage differences

Analogue signals often have to be conveyed from a signal source to a signal destination while a significant voltage difference exists between the grounds. The difference arises from other currents flowing between the systems, or from magnetically induced voltages, and is added in series with any signal. A typical measurement with an oscilloscope readily shows these disturbances on the expected waveform. However, if the probe, or a co-axial lead is correctly used, the ground noise will reduce by a factor of up to a hundred because of a canceling voltage induced in the centre conductor. It is surprising that few engineers and technicians know of the phenomena described in this paper, which so strongly influence their signal measurements.

The predominant effect of co-axial cable in this situation is unrelated to the transmission line equations. The 1MΩ oscilloscope input impedance is far from a matched termination, and a 1.5 m probe is electrically short at the 2 kHz to 2 MHz frequencies where characteristics change significantly. The behaviour depends primarily on the resistance of the screen, and the presence of even short *pigtail leads,* which are unwanted wires connecting the screen of the co-axial cable to the signal's ground reference.

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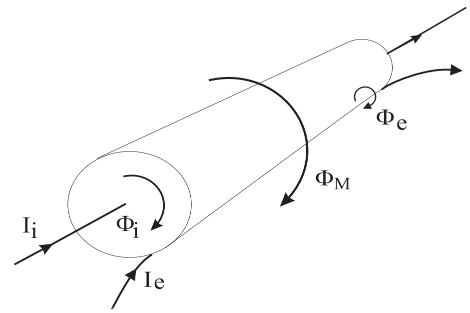

Fig. 1. Flux and current paths.

When correctly used, good co-axial cable will only allow a thousandth of the ground voltage difference for frequencies above 2 MHz to be coupled into the signal, and only one thousandth of the signal current to flow through the low impedance inter-chassis connection. A 1 mm long pigtail lead will double the cross-coupling, so it is essential that electronic packaging designers understand the implications of pigtail leads, and preferably understand the mechanism that makes these so critical.

Differential amplifiers are commonly assumed to be the solution to removing earth voltage differences from signals, but they have limited bandwidth and common mode rejection at frequencies exceeding a few MHz. Knowledge of how co-ax inherently provides the ground voltage correction enables designers to avoid unnecessary circuit complications and retain the simplicity of single mode signals.

B. Arrangement of this paper

In the next section we examine the magnetic fields in, and around a co-axial cable, and the equivalent circuit model that this yields. Section three computes the ground voltage coupling into the signal, and gives measurements that justify model parameters for both co-ax cable and twisted pair cables. Section four examines the split of the returning signal current through the ground and co-ax screen. Section five describes effects of connections at the co-ax cable ends. Section six describes experiments to verify and illustrate the theory, and is followed by the conclusion.

II. Physical model

Figure 1 shows the flux paths in a section of co-axial cable. The flux density between the conductors is only influenced by I_i. At radius r, the flux density is $B_i = \mu_0 I_i / 2\pi r$. The flux density outside the cable is $B_e = \mu_0(I_i + I_e)/2\pi r$, and is zero if I_i and I_e are opposite as is commonly desired.

A. Internal flux and inductance

The internal flux per unit length, Φ_i, is obtained by integrating B_i in the inter-conductor space from r_1 to r_2, giving

$$\Phi_i = \frac{\mu_0 I_i}{2\pi}\ln(r_2/r_1) \tag{1}$$

The internal inductance per unit length is $L_i = \Phi_i/I_i$

$$L_i = \frac{4\pi \times 10^{-7}}{2\pi}\ln(r_2/r_1) \tag{2}$$

$$= 200nH \times \ln(r_2/r_1) \tag{3}$$

The natural logarithm of the *ratio* of the radii results in the inductance per unit length increasing by 0.2μH every time the outer conductor diameter is *multiplied* by 2.72, or the inner conductor diameter is *divided* by 2.72.

B. External flux and screen inductance

Flux Φ_M encircles both conductors, so forms a perfectly coupled transformer with winding inductance equal to the mutual inductance M. Equation 3 shows that the inductance *per unit length* of a straight wire tends to infinity if we allow r_2 to increase without bound. We thus have to use equations for the inductance of a loop or parallel wires [1] to estimate M. Both cases exhibit the same logarithmic dependence on r_1 as equation 3, and predict inductances of near 1 μH/m for 1 mm diameter wire or cable. It is worth noting that M is unaffected by scaling the dimensions of the cable *and* housing. A thin cable which is typically used in small equipment will thus have a similar inductance per unit length to a thick cable which is probably used in larger equipment. The figure of 1 μH/m is thus almost a universal constant, and is typical in simple laboratory measurements.

C. Equivalent circuit

The above magnetic fields lead to the electrical equivalent circuit in figure 2. The resistances R_e and R_i are the physical resistances of the conductors. Inductance L_e is the leakage inductance, and is caused by flux Φ_e encircling only the external conductor. Its most common cause is pigtail leads on the screen, or flux encircling loosely woven strands of the screen. With RG-58 cable fitted with BNC connectors, L_e is typically a thousand times less than M.

The 100 pF/m capacitance of 50Ω co-ax will be ignored in the rest of the paper since for the electrically short situation considered here, it can be treated as an external capacitor. For electrically longer cases, the cable must be terminated and the interference voltage, V_o treated as an EMF.

With a twisted pair cable typically only 60% of the flux encircling the ground conductor encircles the signal conductor. The mutual inductance, M is thus only k (the coupling factor) times the single conductor inductance, while L_e equals the pigtail inductance plus $(1-k)$ times the line inductance.

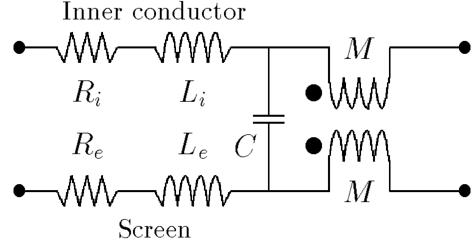

Fig. 2. Co-ax equivalent circuit.

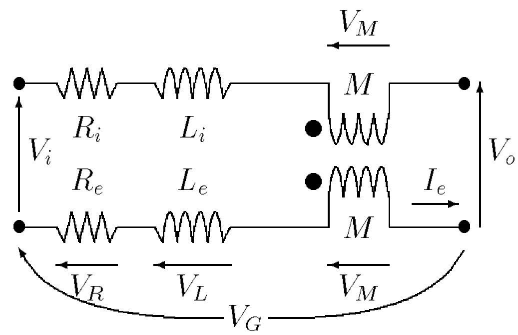

Fig. 3. Ground voltage transfer circuit.

III. Voltage transfer ratio

For reasons soon to be seen, the screen of co-axial cable is usually connected to the ground at the signal source and receiver (load). Figure 3 shows the voltage V_G that is normally found between these two ground points and is generated by other ground currents and magnetic fields.

We assume a high impedance receiver, an electrically short cable, and no input signal, V_i. The received voltage, V_o, is the sum of the voltages over each component in the loop. Note that the mutual inductance induces a voltage in *series* with the signal conductor that *cancels* the voltage appearing across the mutual inductance component in the screen, so that

$$V_0 = V_M + V_L + V_R - V_M \tag{4}$$

The voltage transfer ratio of V_G to V_o is thus

$$T_v = \frac{V_o}{V_G} = \frac{sL_e + R_e}{s(L_e + M) + R_e} \tag{5}$$

This shows us that if the pigtail inductance is zero, the transfer ratio becomes independent of the cable length since the other impedance values are all proportional to length.

The pole in T_v occurs at

$$s_p = \frac{R_e}{L_e + M} \tag{6}$$

which is about 15mΩ/1μH = 16 krad/s or 2.5 kHz. Few engineers believe this figure!

The solid line in figure 4 shows measured ground voltage attenuation ($1/T_v$) for RG-58 cable [2]. It shows the expected break point at 2.5 kHz, the 20dB/decade slope, and a maximum attenuation of 60 dB at 10 MHz.

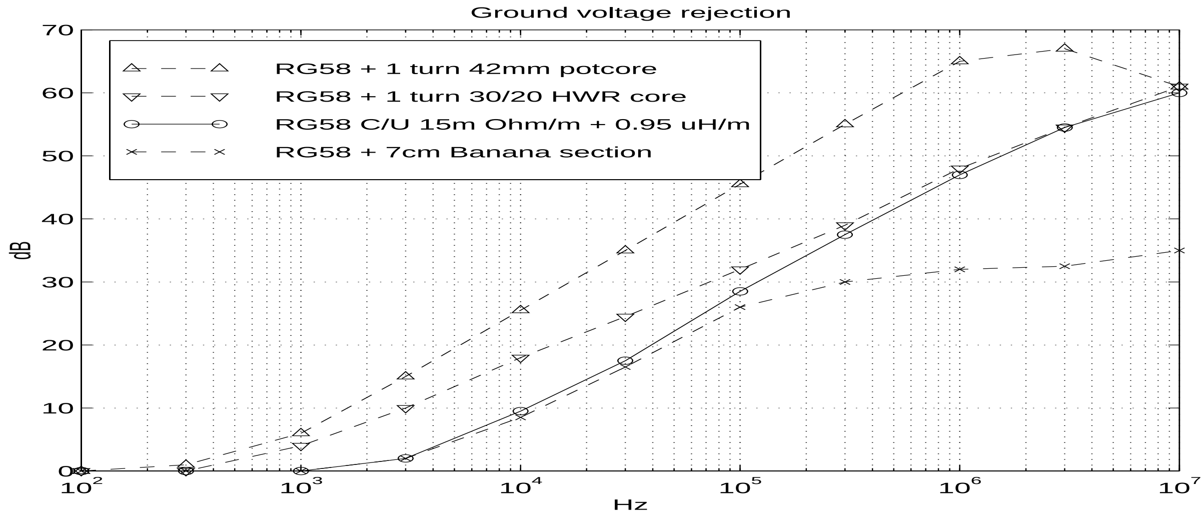

Fig. 4. Ground voltage rejection ($1/T_v$) measurements.

A. Common mode chokes

A common mode choke is produced by fitting a high permeability core around the cable. The reduced reluctance of the magnetic path around the cable increases Φ_M and thus M, and lowers the pole frequency and consequently V_o (see equation 6). Winding multiple turns through the core further increases M.

The top curve in figure 4 shows how T_v improved as a single turn of the co-ax was wound around a 42 mm pot-core. The reduced reluctance to flux encircling the co-ax increases M, lowering s_p to 500 Hz and improving T_v by 16 dB for all frequencies. A large HWR 30/20 C core closed around the co-ax also lowered T_v for frequencies below 100 kHz, but made no improvement above 1 MHz, possibly because its permeability drops and it introduces a loss resistance adding to R_e at high frequencies.

B. Transfer Impedance

The *Transfer Impedance*, Z_t, is defined as V_o/I_e with $V_i = 0$, and can be established experimentally by measuring V_o and I_e. The previous equations show that $Z_t = R_e + sL_e$, which is proportional to cable length. The constancy of the voltage transfer ratio follows from I_e being inversely proportional to cable length if V_G is constant.

The author's fear in using the transfer impedance concept without being familiar with the relations in this paper is that one could conclude that disconnecting one end of a cable's screen would prevent I_e from flowing, thereby making $V_o = I_e V_t$ zero — which will not happen.

Reference [3] gives curves for $|Z_t(f)|$ for various co-axial cables. The RG-58 data match $|R_e + sL_e|$ from 30 kHz to 10 GHz! The value of R_e varies from 5 mΩ/m for the 1 cm diameter RG-8A/U to 40 mΩ/m for the 3 mm diameter RG-174U. L_e has a value of 1.25nH/m with less than 25% variation. It is interesting to note that $|Z_t|$ for semi-rigid co-ax reduces above 100 kHz, and depends strongly on wall thickness. $|Z_t|$ on the double braided RG-55U drops until 10 MHz, whereafter it increases as if it were normal co-ax with a L_e of 16 pH, giving 40 dB lower T_v than RG-58.

The simple model in figure 2 thus represents single braided cables well, and is conservative with better cables. It emphasizes the importance of the pigtail which will be even more significant with better cables such as the semi-rigid types.

C. Pigtail inductance and twisted pairs.

If T_v is measured (with care!) for RG-58 cable with good BNC connectors, it is found to be better than -60 dB above 2 MHz, implying that L_e is less than *one thousandth* the value of M. We saw earlier that the inductance of a single wire is not very sensitive to its parameters, so we can interpret the -60dB figure as implying that the leakage inductance of a metre of RG-58 is equivalent to less than *one millimeter* of pigtail lead! By connecting the screen of a one metre length of RG-58 using a one millimeter length of wire or printed circuit board track, we this *double* L_e and cause the earth voltage transfer ratio to worsen by 6 dB, thus doubling the coupled high frequency interference.

The lower curve in figure 4 shows what happened when the co-ax was broken with adapters transitioning to banana connectors at the standard spacing, and returning to co-ax. The banana section length was 7cm, with an estimated inductance of 29nH. T_v increased dramatically for frequencies above 100 kHz, and was limited to -35 dB. This 25 dB loss in T_v is caused by the increase in L_e that occurs when the screen no longer surrounds the signal conductor.

A twisted pair cable has an even higher value of L_e since no attempt is made to surround the signal wire with the ground lead, leading to ground voltage attenuations of as little as 12 dB.

D. Pigtails ground leads on oscilloscope probes

An oscilloscope probe is a common example of co-axial cable used in the laboratory. The co-axial struc-

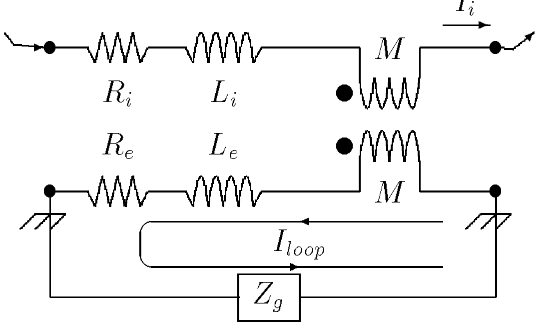

Fig. 5. Induced loop current.

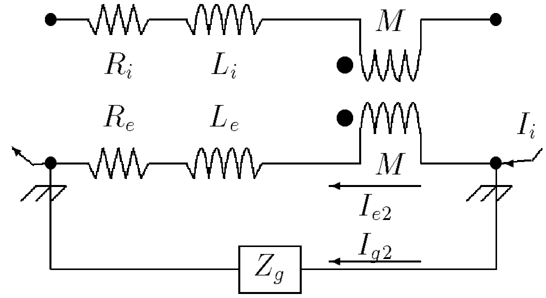

Fig. 6. Splitting of return current.

ture of the probe and the cable is used to cancel the earth voltage differences between the oscilloscope and the system under test. A good probe will maintain a co-axial structure right up to the signal pin. The removable tip must expose the central signal pin, which *must* emerge from a ground ring. The ring must be able to be pressed into contact with a ground while the central pin contacts the test signal. It is only by grounding the ring without any intervening pigtail lead that the full earth voltage rejection of 60dB will be realised.

The 'flying ground lead' attached to most probes is a poor ground, being a long pigtail that typically limits earth voltage rejection to 20dB. Some cheap probes don't even have the grounding ring, which raised severe doubts about the probe's construction inside the molded plastic!

E. Cable trays

A conductive cable tray bonded at both ends to equipment grounds will provide a reasonable level of ground voltage cancellation to cables lying close to its metallic surface, or deep inside the tray. The flux Φ_M is the flux encircling the cable tray, while Φ_e is the flux passing between the cable and the inside of the tray. If a cable lies deeply within a cable tray, Φ_e becomes much smaller than Φ_M, enabling moderate values of earth voltage cancellation to be achieved.

IV. Ground current transfer function

It is often believed that all of the return current automatically flows on the inside of the screen of the co-ax. Since the source and load are grounded, possibly via lower impedance paths than the co-ax, this clearly will not occur at DC. We now explore the real situation.

We start with the situation in figure 5 where the signal current I_i is fed through the inner conductor, but not returned via the screen which is grounded at both ends. These are connected through a ground impedance of $Z_g(s)$.

The signal current and mutual inductance induce a voltage sI_iM in the screen, causing a current of

$$I_{loop} = I_i \frac{sM}{Z_g(s) + R_e + s(M + L_e)} \tag{7}$$

to flow in a loop through the ground and the cable's screen. Note that I_{loop} flows in the normal re-

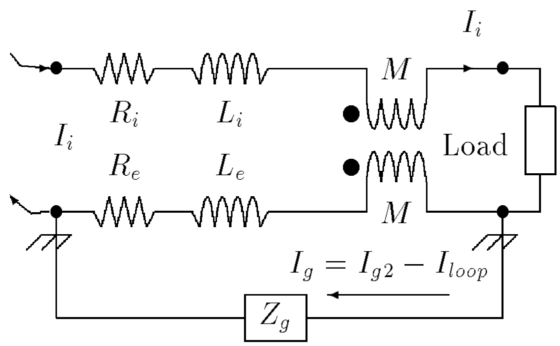

Fig. 7. Ground current = sum of loop and portion of returning signal current.

turn direction for current in the screen, but in the *opposite* direction to currents returning via the ground impedance.

Now consider figure 6, where we pass I_i through the ground, without passing it through the inner conductor. The current splits into I_{e2} and I_{g2} with

$$I_{g2} = I_i \frac{R_e + s(L_e + M)}{Z_g(s) + R_e + s(L_e + M)} \tag{8}$$

In figure 7 we revert to the normal situation and pass I_i through the inner conductor, the load, and the screen. The ground current, I_g, is now the sum of the current in the previous two cases, taking the directions into account.

$$\begin{aligned} I_g &= I_{g2} - I_{loop} && (9)\\ &= I_i\Big(\frac{R_e + s(L_e + M)}{Z_g(s) + R_e + s(L_e + M)} && (10)\\ &\quad - \frac{sM}{Z_g(s) + R_e + s(L_e + M)}\Big) && (11) \end{aligned}$$

The *ground current transfer function* is thus

$$T_i = \frac{I_g}{I_i} = \frac{R_e + sL_e}{Z_g(s) + R_e + s(L_e + M)} \tag{12}$$

which is identical to T_v in equation 5 if $Z_g(s)$ is zero. In practice, equipment should be well grounded, so $Z_g(s)$ is likely to be smaller than $R_e + sM$, leading to litte error if one uses the curve for T_v in place of T_i.

At DC, only the fraction of the signal current given by $R_e/(R_e + Z_g(0))$ flows through the ground connection. As the frequency increases, the induced current increasingly cancels the current returning through the ground connections.

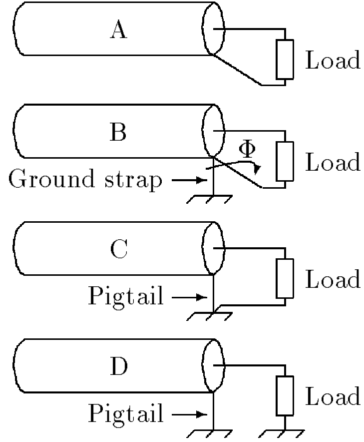

Fig. 8. Four grounding configurations for co-ax at the load end.

The significance of the above relations for T_i is that in spite of possibly zero impedance ground connections between co-ax cable ends, the ground current starts falling at 6dB per octave from about 2 kHz to a level of about -60dB, provided that no pigtails are used. Since the external magnetic field is only caused by the current flowing through the ground (co-ax conductor fields cancel) the field generated by the cable can be 60 dB below that of a non co-axial path. The importance of keeping pigtails of a 1 m co-ax to less than 1 mm length is apparent. As an example, a 1 cm pigtail will increase the field from -60 dB to -40 dB — a 20 dB increase!

A useful consequence of the above theory is the realization that power supply leads carrying fast pulsed current can be connected using co-ax. The co-ax will prevent the pulsating return current from passing through the other grounding system and injecting interference in many other signals.

An alternative analysis to that presented above is to assume that the ground current, I_g flows in a loop through the ground path and the screen in the same direction as I_{loop}, and I_i returns in the conventional direction through the screen.

Summing voltages around the loop produces

$$(-I_i + I_g)(R_e + sL_e + sM) + sMI_i + Z_g = 0 \quad (13)$$

which reduces to equation 12. This short analysis hides the reality of the induced loop current illustrated in figure 5.

V. Grounding connections

Figure 8 shows four possible connections from the co-ax to the receiver.

- Option A only provides capacitive screening for the signal. A transformer or differential input stage is required to reject ground voltage differences. The common mode rejection ratio of the input stage must exceed 60dB above 2 MHz if it is to compete with the other connection options above 2 MHz.
- Option B grounds the screen which zeros its voltage, and enables the screen to inductively cancel high frequency ground voltage differences from the signal. Noise current flowing through the ground strap will also create magnetic fields that should not be allowed to pass through the loop of the load or differential receiver, else ground noise will be re-injected.
- Option C uses a single-ended input stage and relies on the co-ax to reject the ground signal. The rejection is limited by the inductance of the pigtail lead, which should be kept shorter that a thousandth of the cable length for optimal results.
- Option D is similar to C, but any ground voltage differences between the end of the pigtail and the load will add to the signal.

VI. Experiments

This section describes how some characteristics of co-ax can be measured or demonstrated.

A. Measurement of small resistance and inductance

Most laboratories have a function generator with 50 Ω output impedance and capable of a 10 kHz 12.5 V peak-peak (unloaded) triangular output. Such a generator connected via 50 Ω co-axial cable to a low impedance, $R + sL$ develops a voltage across it that is a triangular waveform of 0.25 R Volts p-p added to a square wave of Lf Volts p-p, where f is the frequency in Hz. For f = 10kHz, the 20 mΩ and 1 μH of a 1 m length of 20 AWG wire produces a 5 mV triangular component and a 10 mV square component. These are readily seen and identified on an oscilloscope[1]. Raising the frequency to 1 MHz causes the inductively produced square waveform to increase which enables the inductance of a 1 cm length of wire (about 10 nH) to be verified from the 5 mV square component it produces. Earlier lessons on grounding need to be applied to make the measurement correctly.

B. Non-reciprocal transformer action?

Experiments with co-ax always stimulate thought! Those that doubt if the transformer action implied by M really occurs, should connect a 1 MHz function generator to the ends of the screen of a length of co-axial cable, and measure the voltage appearing between the ends of the inner conductor. The voltages will appear identical on an oscilloscope. However, if the function generator is connected to the inner conductor and the oscilloscope is connected to the ends of the screen, only about 70% of the voltage is measured. The reason is that L_e and M form an inductive voltage divider, and only the voltage across M is induced on the screen. In the first connection, L_e is in series with the oscilloscope input and has no effect!

C. T_v and Z_t measurements

Figure 9 shows how T_v may be measured. The figure shows a length of RG-58 co-ax with BNC connectors on its ends. The 50 Ω termination or a short circuit ensures there is zero differential input voltage. The lower voltmeter or oscilloscope measures the ground

[1]Measuring technique popularized at Stellenbosch by F.E. du Plessis.

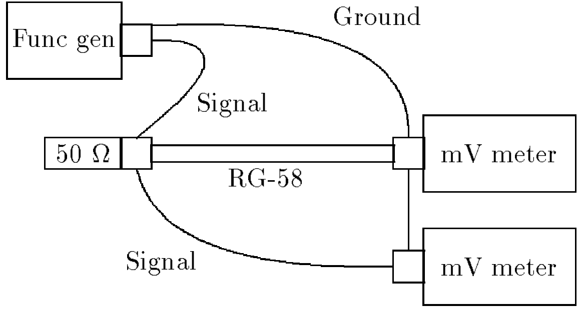

Fig. 9. Test method for ground voltage coupling.

voltage between cable ends, while the top voltmeter measures the received voltage.

Note that the function generator is short-circuited by the screen's low resistance and inductance. The current into the screen is nearly constant till 5 MHz but its terminal voltage is not! The signal voltage emerging from the co-ax cable is thus nearly independent of frequency (since Z_t only starts increasing above about 5 MHz). T_v is thus seen from the increase with frequency of the voltage appearing between the ends of the screen!

D. Oscilloscope probe grounding

If the flying ground lead of an oscilloscope probe is earthed on circuit board carrying 10 MHz clock signals, and the probe tip is connected to the circuit earth, much 'fuzz' will be seen. If the probe tip is removed, and the grounding ring (if it is there!) is contacted directly on the circuit earth, all of the 'fuzz' should disappear. Since the fuzz is high frequency noise, the co-axial structure in the probe and it's co-axial cable will be able to cancel ground voltages to a level of close to -60 dB. Thus even a 1V ground bounce will induce only a 1 mV signal distortion, making it easy to make accurate measurements. The important thing to note is that 10 mm of ground lead will increase the fuzz by a factor of ten with a typical probe. Ground is thus ground, and not 5 mm away from it!

E. Ground current extraction

A 2 m cable was used to connect a function generator to a 50 Ω termination, and the casings of the BNC connectors at either end of the cable were connected together by a 15 cm length of 'ground' wire with resistance of 11 mΩ. A current probe was fastened about the ground wire, and the ground current measured. Below 100 Hz, the ground current was about 70% of the signal current, whereafter it reduced as per equation 12, reaching at least -50 dB at 1 MHz, at which point it could no longer be measured reliably. This experiment confirms the mechanism described by equation 12, whereby ground currents increasingly return via the co-ax screen, in spite of other lower impedance ground paths.

F. Controlled and low inductance

A fast clock driver for a high capacitance CCD was needed, and had to be on a separate but close-by circuit board. The lead inductance formed a part of the resonant drive circuit, and had to be kept independent of wire routing. The internal inductance of a co-axial feed was used in the energy recovery drive circuit, and remained constant, in spite of variations in wiring harness dressing. Some thought also shows that the inductance of the screen of the co-ax does not appear in series with the load since its voltage drop is canceled by an EMF induced in it by the signal current and the mutual inductance, M. Another argument is that if the return current largely flows via the screen of the co-ax, its external field is near zero, so the inductance must have little effect. These ideas and insights are due to [4].

G. Twisted pairs and mains cabtyre

The previous theory applies directly to twisted pairs if one uses the correct figures for L_e, M, and R_e. The twisting hardly influences these parameters, and is primarily used to reduce coupling of external magnetic fields. It is interesting to record the parameters for standard three-core mains cabtyre, which were: $L_e = 0.4\mu$H/m, $M = 0.84\mu$H/m, and R=13mΩ/m. This yields a coupling factor of 0.74, and a value for ω_p of 1.8 kHz. This coupling coefficient is similar to many twisted pairs.

VII. Conclusion

This paper has described a lumped equivalent model of earth voltage coupling into co-axial cable that is applicable in most laboratory applications. Since developing this model in 1973, the author has found it to be valuable in explaining and predicting ground signal coupling in practical application of coaxial cables.

The theory has been published independently by others such as [5]. It is not however fashionable in the EMC community, which seems to prefer a common and differential mode approach. This author believes however that the simple lumped model described here is the best means of explaining exactly how co-axial cable works in most situations, including oscilloscope probes and interconnections. It also gives a simple justification of the great importance of controlling pigtail leads as signals enter or exit the co-axial structure.

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